5.5 Critical Flow Calculations

5.5.1 Background

In the design of open channels, it is important to calculate the critical depth in order to determine if the flow in the channel will be subcritical or supercritical. If the flow is subcritical it is relatively easy to handle the flow through channel transitions because the flows are tranquil and wave action is minimal. In subcritical flow, the depth at any point is influenced by a downstream control, which may be either the critical depth or the water surface elevation in a pond or larger downstream channel. In supercritical flow, the depth of flow at any point is influenced by a control upstream, usually critical depth. In addition, the flows have relatively shallow depths and high velocities.

Critical depth depends only on the discharge rate and channel geometry. The general equation for determining critical depth is expressed as:

\[
\frac{Q^2}{g} = \frac{A^3}{T}
\]  

(Eq. 5.5.1-1)

Where:
- \( Q \) = discharge rate for design conditions (cfs)
- \( g \) = acceleration due to gravity (32.2 ft/sec^2)
- \( A \) = cross-sectional area (ft^2)
- \( T \) = top width of water surface (ft)

Note: A trial and error procedure is needed to solve equation 5-6.

5.5.2 Semi-Empirical Equations

Semi-empirical equations (as presented in Table 5.5.2-1) or section factors (as presented in Figure 5.5.2-1) can be used to simplify trial and error critical depth calculations. The following equation is used to determine critical depth with the critical flow section factor, \( Z \):

\[
Z = \frac{Q}{(g^{0.5})}
\]  

(Eq. 5.5.2-1)

Where:
- \( Z \) = critical flow section factor
- \( Q \) = discharge rate for design conditions (cfs)
- \( g \) = acceleration due to gravity (32.3 ft/sec^2)

The following guidelines are given for evaluating critical flow conditions of open channel flow:
5.5.2.1 A normal depth of uniform flow within about 10 percent of critical depth is unstable and should be avoided in design, if possible.

5.5.2.2 If the velocity head is less than one-half the mean depth of flow is subcritical.

5.5.2.3 If the velocity head is equal to one-half the mean depth of flow, the flow is critical.

5.5.2.4 If the velocity head is greater than one-half the mean depth of flow, the flow is supercritical.

The Froude number, Fr, calculated by the following equation, is useful for evaluating the type of flow conditions in an open channel:

\[ Fr = \frac{v}{(gA/T)^{0.5}} \]  

(Eq. 5.5.2.4-1)

Where:  
Fr = Froude number (dimensionless)  
v = velocity of flow (ft/s)  
g = acceleration of gravity (32.2 ft/sec^2)  
A = cross-sectional area of flow (ft^2)  
T = top width of flow (ft)

If Fr is greater than 1.0, flow is supercritical; if it is under 1.0, flow is subcritical. Fr is 1.0 for critical flow conditions.
<table>
<thead>
<tr>
<th>Channel Type</th>
<th>Semi-Empirical Equations for Estimating Critical Depth</th>
<th>Range of Applicability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Rectangular</td>
<td>$d_c = \left[Q^2/(gb^2)\right]^{1/3}$</td>
<td>N/A</td>
</tr>
<tr>
<td>2. Trapezoidal</td>
<td>$d_c = 0.81\left[Q^2/(gz^{0.75}b^{1.25})\right]^{0.27} - b/30z$</td>
<td>$0.1 &lt; Q/b^{2.5} &lt; 0.4$ For $0.5522 &lt; Q/b^{2.5} &lt; 0.1$, use rectangular channel equation</td>
</tr>
<tr>
<td>3. Triangular</td>
<td>$d_c = [2Q^2/(gz^2)]^{1/5}$</td>
<td>N/A</td>
</tr>
<tr>
<td>4. Circular</td>
<td>$d_c = 0.325(Q/D)^{2/3} + 0.083D$</td>
<td>$0.3 &lt; d_c/D &lt; 0.9$</td>
</tr>
<tr>
<td>5. General</td>
<td>$(A^3/T) = (Q^2/g)$</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Where: $d_c$ = critical depth (ft)  
$Q$ = design discharge (cfs)  
$g$ = acceleration due to gravity (32.3 ft/s$^2$)  
$b$ = bottom width of channel (ft)  
$z$ = side slopes of a channel (horizontal to vertical)  
$D$ = diameter of circular conduit (ft)  
$A$ = cross-sectional area of flow (ft$^2$)  
$T$ = top width of water surface (ft)  

1See Figure 5-5 for channel sketches  
2Assumes uniform flow with the kinetic energy coefficient equal to 1.0  
3Reference: French (1985)  
4Reference: USDOT, FHWA, HDS-4 (1965)  
5Reference: Brater and King (1976)
Figure 5.5.2-1
Open Channel Geometric Relationships For Various Cross Sections

<table>
<thead>
<tr>
<th>Section</th>
<th>Area</th>
<th>Top Width</th>
<th>Water Perimeter Hydraulic Radius</th>
<th>Critical Depth Factor</th>
<th>Note:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>(bd + \sqrt{d^2 + (b + 2d)^2})</td>
<td>(b + 2d)</td>
<td>(b + 2d)</td>
<td>(\frac{2d}{\sqrt{b + 2d}})</td>
<td></td>
</tr>
<tr>
<td>Triangle</td>
<td>(\frac{3}{2}d^2)</td>
<td>(d)</td>
<td>(d)</td>
<td>(\frac{2dT}{3\pi})</td>
<td></td>
</tr>
<tr>
<td>Parabola</td>
<td>(\frac{1}{12}d^3)</td>
<td>(\frac{2d}{3})</td>
<td>(\frac{2d}{3})</td>
<td>(\frac{\pi D(360 - \theta)}{360})</td>
<td></td>
</tr>
<tr>
<td>Circle-Circle Full</td>
<td>(\frac{1}{3}d^3)</td>
<td>(\frac{2d}{3})</td>
<td>(\frac{2d}{3})</td>
<td>(\frac{\pi D(360 - \theta)}{360})</td>
<td></td>
</tr>
</tbody>
</table>

- \(D\) = Depth
- \(T\) = Slope Horizontal Distance
- \(L\) = Critical Depth Section Factor

Note: Small \(L\) = Side Slope Horizontal Distance
Large \(L\) = Critical Depth Section Factor

When \(\theta > 0.25\), use \(\theta/100\) in above equations.

Reference: USDA, SCS, NEH 6 (1886).
END OF SECTION 5.5